Modeling and problem solving with Xpress-Mosel

FICO Xpress Training

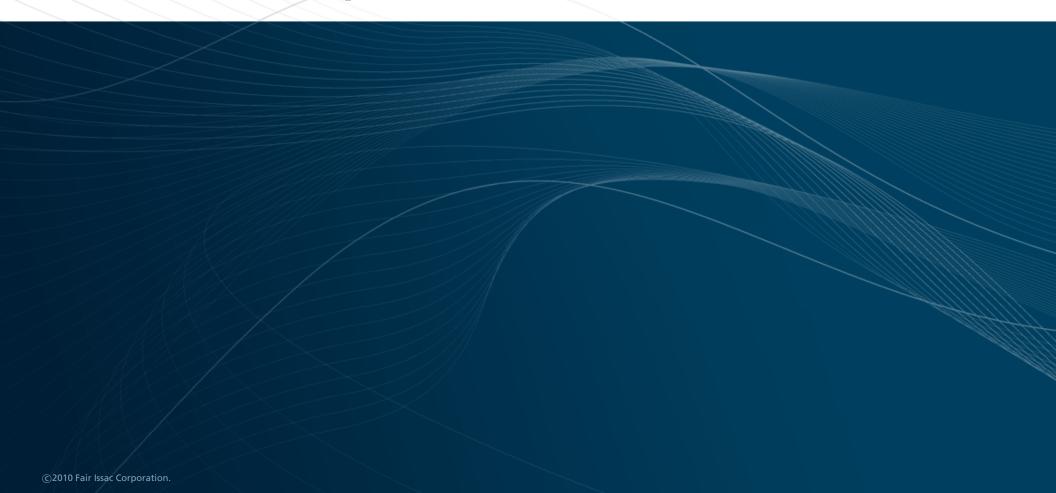
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Introduction, Xpress overview







- » Introduction to Xpress
- » Modeling with Mosel:
 - » Linear and Mixed Integer Programming (LP and MIP)
 - » Accessing data sources
 - » Programming language features
- » Embedding models in applications





» At the end of the course you will

- » be familiar with optimization methods and the terminology used to describe them
- » be confident about formulating optimization models and understanding the solution
- » know how use Xpress to model and solve problems
- » be able to embed a model in a application



- » Not a replacement for the reference manuals!
- » Focuses on areas that are of practical importance
- » Does not try to be exhaustive
 » Pointers to reference material at the end of every chapter



Overview of Xpress

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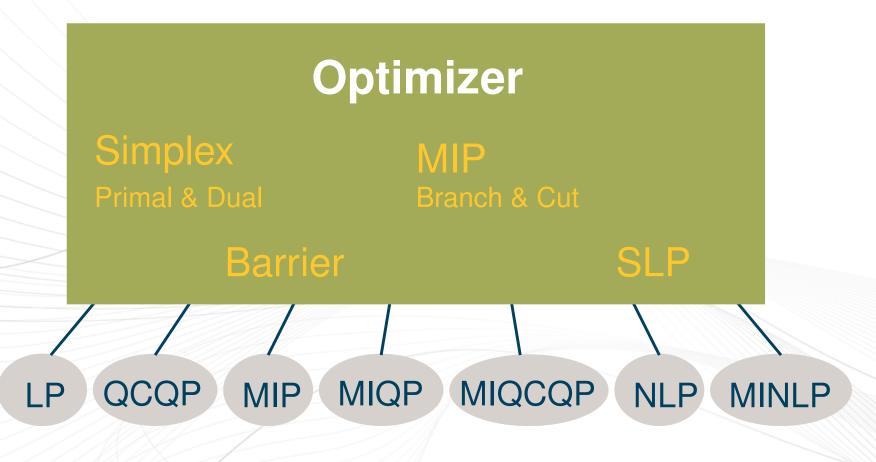
» Optimization algorithms

- » enables you to solve different classes of problems
- » built for speed, robustness and scalability

» Modeling interfaces

- » enables you to provide your problem in the most suitable way for your application
- » built for ease of use and interfacing







» Mosel

- » formulate model and develop optimization methods using Mosel language / environment
- » BCL
 - » build up model in your application code using object-oriented model builder library
- » Optimizer
 - » read in matrix files
 - » input entire matrix from program arrays





» A modeling and solving environment » integration of modeling and solving programming facilities >> open, modular architecture » Interfaces to external data sources (e.g. ODBC, host application) provided » Language is concise, user friendly, high level » Best choice for rapid development and deployment



» Mosel language: to implement problems and solution algorithms
 ⇒ model or Mosel program
 » Mosel Model Compiler and Run-time Libraries: to compile, execute and access models from a programming language
 ⇒ C/C++, C#, Java, or VB program



» Mosel Native Interface (NI): to provide new or extend existing functionality of the Mosel language

 \Rightarrow module

» Xpress-IVE: graphical user interface, representation of the problem matrix, solution status/progress graphs, and result display



- » Embed Mosel models directly in your application
- » Access the solution within your application
- » Compiled models are platform independent
 » Enjoy benefits of structured modeling language and rapid deployment when building applications
- » Available for C, Java, C#, and VB





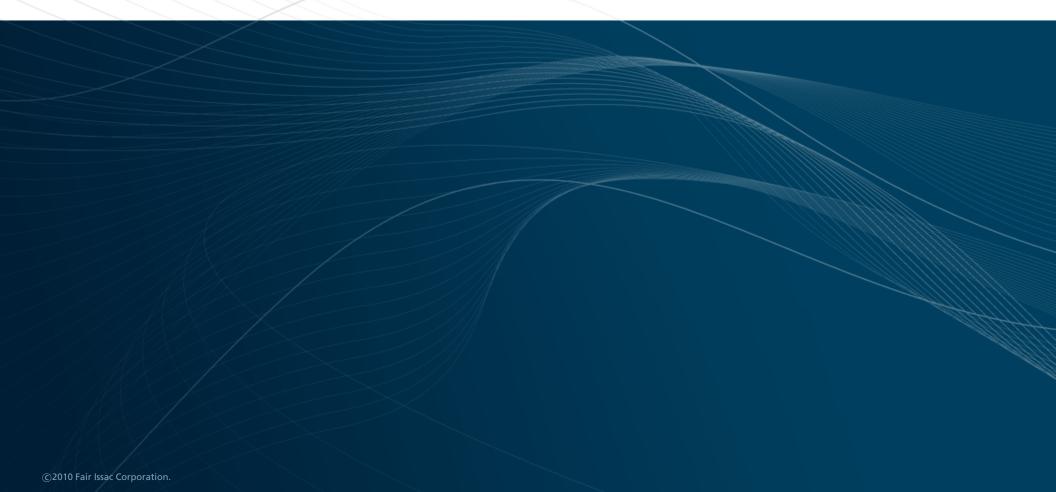
- » Visual Studio style visual development environment for optimization & model building with Mosel
 » Mosel model editor & compiler
 » Real time graphs show optimization
 - performance
- » Browse solution values in entity tree

» Stand-alone command line executables with text interfaces

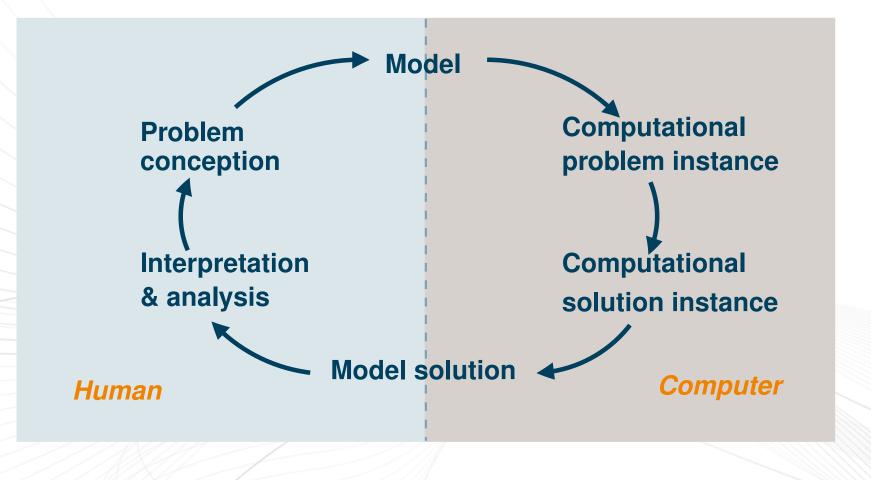
- » Useful for simple deployment using batch/script files
- » Available for all platforms supported by Xpress



Why use modeling software?









- » Developing a working model is the difficult bit
- » Important to have software that helps
 - » speed to market
 - » verify correctness
 - » maintenance & modification
 - » algorithmic considerations
 - » execution speed



- » The concepts we describe how to formulate and solve problems – apply to all modeling software
- » In this course we will use the Xpress-IVE development environment with the Xpress-Mosel language because it is
 - » easy to understand and learn
 - » easy to use



Xpress-IVE demonstration





» Models: new, saving, opening, switching

- start a new model
- open an existing model
- save current model
- show list of available modules
- » Bars: editor, entity, info, output (run)
 - switch between window layouts



» Editor: colors, auto-complete, tool tips

- copy selection
- cut selection
- paste selection
- **go to next / last line with same indentation**
- go to previous / next cursor position (line)
- undo / redo last editor command

Xpress-IVE demonstration



» Compile, run

- compile current model
- execute current model
- open run options dialog
- pause execution
- interrupt execution
- search for the N best solutions
- start infeasibility repair



» Output bar: log, stats, matrix, graphs, tree
» Viewing solution values
» Problem and matrix export and import
generate BIM file
is export the problem matrix
optimize an imported matrix



- » Search, bookmark
 - search
 - delete bookmarks
- » Help
 - help
 - model generation wizzard & example models
 module generation wizzard



» Debugger

- set/delete breakpoint at cursor
- define conditional breakpoint
- start/stop debugger
- step over an expression
- step into an expression
- run up to the cursor
- show debugger options dialog
- » Profiler
 - start the profiler



- » The manual Getting Started with Xpress introduces first time or occasional users to modeling with Mosel and BCL, or the direct Optimizer interface
- » The Evaluators Guide and Advanced Evaluators Guide provide a quick walk-through of the Getting Started examples and some more advanced features



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Modeling with Mosel

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Overview



- » Modeling basics
- » Accessing data sources
- » Advanced modeling topics
- » Programming language features
- » Mosel modules and packages

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Modeling basics

» A first model
» Data structures and loops
» Model building style





» Definition of decision variables and constraints
 » Solving with Xpress-Optimizer
 » Solution output



- » A joinery makes two different sizes of boxwood chess sets.
- » The small set requires 3 hours of machining on a lathe, and the large set requires 2 hours. There are 4 lathes with skilled operators who each work a 40 hour week.
- » The small chess set requires 1 kg of boxwood, and the large set requires 3 kg. Only 200 kg of boxwood can be obtained per week.



- » Each of the large chess sets yields a profit of \$20, and one of the small chess sets has a profit of \$5.
- » How many sets of each kind should be made each week so as to maximize profit?



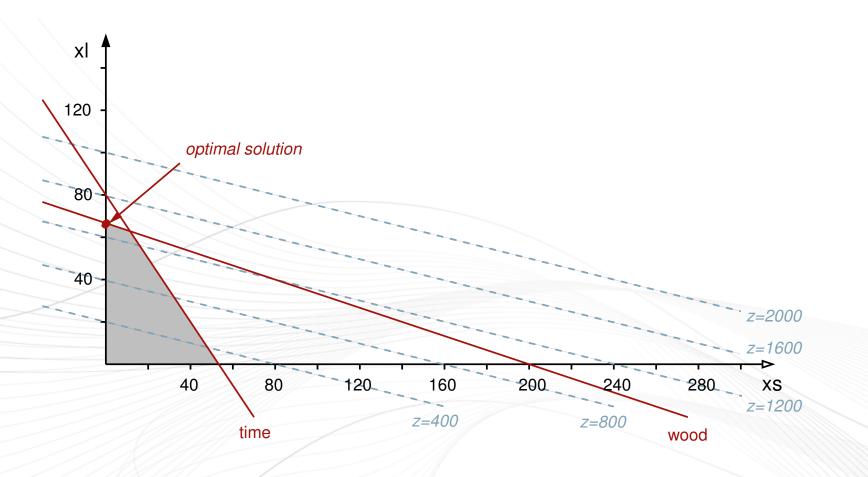
» xl – quantity of large chess sets made
 xs – quantity of small chess sets made

 $\begin{array}{ll} \max & z = 5 \cdot xs + 20 \cdot xl \\ \text{s.t.} & 3 \cdot xs + 2 \cdot xl \leq 160 (= 4 \cdot 40) & \text{(lathe time)} \\ & xs + 3 \cdot xl \leq 200 & \text{(wood)} \\ & xs \cdot xl \geq 0 \end{array}$

xs, $xl \ge 0$

Chess problem: Graphical solution







model "Chess 1"	
uses "mmxprs"	! Use Xpress-Optimizer for solving
declarations	
xs: mpvar	! Number of small chess sets
xl: mpvar	! Number of large chess sets
end-declarations	
3*xs + 2*xl <= 160	! Constraint: limit on working hours
xs + 3*xl <= 200	! Constraint: raw mat. availability
maximize(5*xs + 20*xl)	! Objective: maximize total profit
end-model	

Starting and ending a Mosel model

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model "Chess 1"

•••• end-model



- » uses statement: Say we will use the Xpress-Optimizer library, so that we can solve our problem
- » Options:
 - » noimplicit: force all objects to be declared
 - » explterm: Use ';' to mark line ends

uses 'mmxprs' options noimplicit options explterm



- » mpvar means mathematical programming variable or decision variable
- » Decision variables are unknowns: they have no value until the model is run, and the optimizer finds values for the decision variables



 In optimization problems, decision variables are often just called variables
 In computer programs, a variable can be used to refer to many different types of objects

» For instance, in Mosel models, a program variable can be used to refer to a decision variable, as well as integers, reals, etc.



 » Variables can take values between 0 and infinity by default
 » Other bounds may be specified

x <= 10 y(1) = 25.5 y(2) is_free z(2,3) >= -50 z(2,3) <= 50</pre>





» Have type linctr – linear constraint

declarations
 Wood: linctr
 Inven: array(1..10) of linctr
 end-declarations



 The 'value' of a constraint entity is a linear expression of decision variables, a constraint type, and a constant term
 Set using an assignment statement

Wood := $xs + 3 * xl \le 200$



» An objective function is just a constraint with no constraint type

declarations
 MinCost: linctr
end-declarations

MinCost := 10 * x (1) + 20 * x (2) + 30 * x (3) + 40 * x (4)

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» Generate the matrix and solve the problem:

minimize(MinCost)
maximize(5*xs + 20*xl)

» Load the matrix:

loadprob(MinCost)

» Matrix export:

exportprob(0, "explout", MinCost)



» Can access and manipulate the solution values within the model

writeln('Solution: ', getobjval)

writeln('xs = ', getsol(xs))
writeln('xl = ', getsol(xl))

write('Wood: ', getact(Wood), ' ')
writeln(getslack(Wood))

» Solution values of constraints activity value + slack value = RHS



- » Execute the model chess1.mos.
- » Add printing of the solution values.
- » Is the solution realistic/desirable?
- » Constrain the variables to take integer values only.
- » Add output of constraint activity and slack values.



» Executing model chess1.mos with IVE:

- » double click on the model file to start IVE or open the file from within IVE
- » click on the run button: 🗈

» Model execution from the command line:

mosel -c "exe chess1.mos"

» or:

```
mosel
exe chessl.mos
quit
```

Solution: Completed model Chess 1



model "Chess 1 (completed)"
uses "mmxprs"

! Use Xpress-Optimizer for solving

declarations
 xs,xl: mpvar
end-declarations

! Decision variables

Profit:= 5*xs + 20*xl	! Name the objective function
Time:= 3*xs + 2*xl <= 160	! and the constraints
Wood:= xs + 3*xl <= 200	
xs is integer; xl is integer	! Integrality constraints

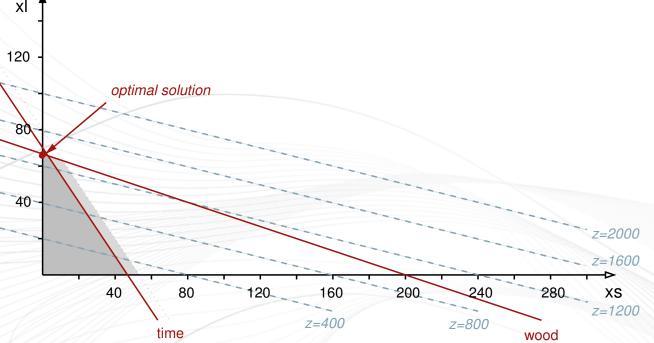
```
maximize (Profit)
```

! Objective: maximize total profit

writeln("Solution: ", getobjval) ! Print objective function value writeln("xs: ", getsol(xs), " xl: ", getsol(xl)) ! Print sol. val.s write("Time: ", getact(Time)) ! Constraint activity writeln(" ", getslack(Time)) ! and slack end-model



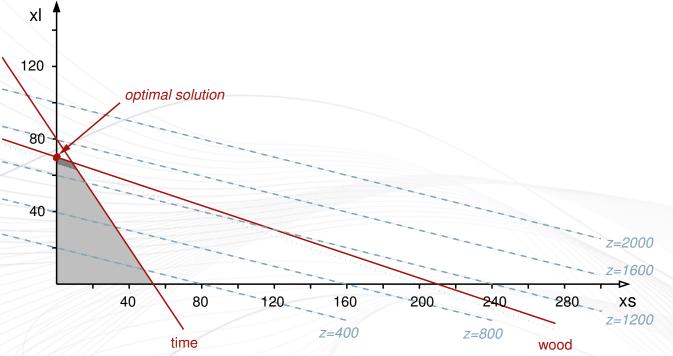
» What happens if machines operate 35 instead of 40 hours?



» Calculate spare capacity: getslack, getactivity



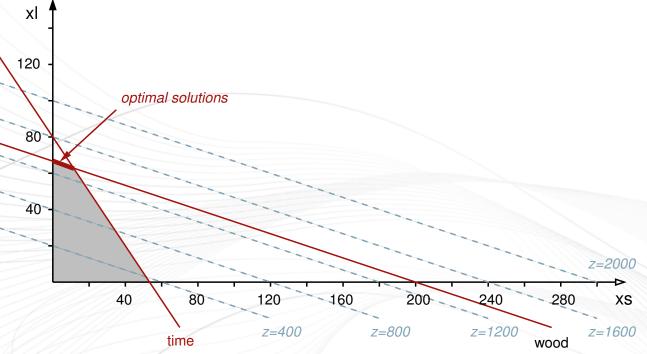
» What is the cost of an extra unit of wood/extra working hour?



» Reduced cost: getrcost



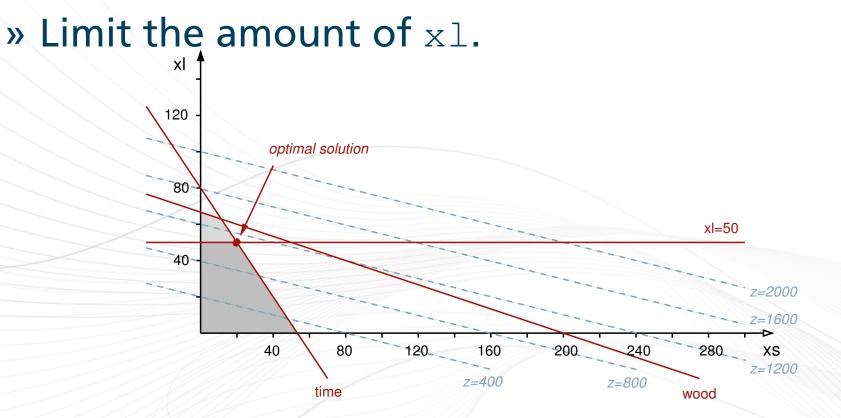
» What is the cost of producing an additional unit of each product?



» Dual values ('shadow prices'): getdual
» Increase price of x1 to reach break even point

Solution analysis





Extending the example: Model Chess 2



uses "mmxprs" options explterm

! Use ';' to mark line ends

declarations

Allvars: set of mpvar; DescrV: array(Allvars) of string; ! Descriptions of variables xs,xl: mpvar; end-declarations

! Set of variables

DescrV(xs) := "Small"; DescrV(xl) := "Large";

```
Profit:= 5*xs + 20*xl;
Time:= 3*xs + 2*xl <= 160;
Wood:= xs + 3 * x1 <= 200;
xs is_integer; xl is_integer;
```

```
! Objective function
! Constraints
```

```
maximize(Profit);
writeln("Solution: ", getobjval);
forall(x in Allvars) writeln(DescrV(x), ": ", getsol(x));
```



- » Set: unordered collection of objects of the same type
 - » used as index sets
 - » special type range sets (= interval of integers)
- » Array: multidimensional table of objects of the same type
 - » used for data, decision variables, constraints
 - » may be dynamic or static

Arrays and loops: Model Chess 3



uses "mmxprs"

```
declarations
R = 1..2
DUR, WOOD, PROFIT: array(R) of real
x: array(R) of mpvar
end-declarations

! Index range
! Coefficients
! Array of variables
```

DUR :: [3, 2] WOOD :: [1, 3] PROFIT :: [5, 20]

```
sum(i in R) DUR(i)*x(i) <= 160
sum(i in R) WOOD(i)*x(i) <= 200
forall(i in R) x(i) is_integer
maximize(sum(i in R) PROFIT(i)*x(i))
writeln("Solution: ", getobjval)</pre>
```

! Constraint definition

! Initialize data arrays

Data declaration



declarations
NWEEKS = 20
DATA_DIR = 'c:/data'
NPROD: integer
SCOST: real
DIR: string
IF_DEBUG: boolean

PRODUCTS = {"P1", "P2", "P4"}
S: set of integer
R: range
COST: array(1..3,1..4) of real
end-declarations

! Integer constant ! String constant ! Integer variable ! Real variable ! String variable ! Boolean variable

! Constant set of string
! Variable set of integer
! Range of integers
! Array of real



NPROD:= 50
SCOST:= 5.4
DIR:= 'c:/data'
IF_DEBUG:= true



» Sum up an array of variables in a constraint

Ctr1:= sum(p in 1..10) (RES(p) * buy(p) + sell(p)) <= 100

Ctr2:= sum(p in PRODUCTS) (buy(p) + sum(r in 1..5) make(p,r)) <= 100

Ctr3:= sum(p in 1..NP) (2*CAP(p)*buy(p)/10 + SCAP(p)*sell(p)) <= MAXCAP





» Use a loop to assign an array of constraints

forall(t in 2..NT) Inven(t):= bal(t) = bal(t-1) + buy(t) - sell(t)





» Use do/end-do to group several statements into one loop

```
forall(t in 1..NT) do
MaxRef(t) := sum(i in PRODUCTS)
use(i,t) <= MAXREF(t)</pre>
```

```
Inven(t) := store(t) = store(t-1) + buy(t) - use(t)
end-do
```





» Can nest forall statements

```
forall(t in 1..NT) do
   MaxRef(t):= sum(i in 1..NI) use(i,t) <= MAXREF(t)</pre>
```

```
forall(i in 1..NI)
    Inven(i,t):= store(i,t) = store(i,t-1) + buy(i,t) - use(i,t)
end-do
```





» May include conditions in sums or loops

forall(c in 1..10 | CAP(c)>=100.0)
MaxCap(c):=
 sum(i in 1..10, j in 1..10 | i<>j)
 TECH(i,j,c)*x(i,j,c) <= MAXTECH(c)</pre>



- » Can extend over several lines and use spaces
- » However, a line break acts as an expression terminator
- » To continue an expression, it must be cut after a symbol that implies continuation (*e.g.* + - ,)



» You should aim to build a model with sections in this order

- » constant data: declare, initialize
- » all non-constant objects: declare
- » variable data: initialize / input / calculate
- » decision variables: create, specify bounds
- » constraints: declare, specify
- » objective: declare, specify, optimize



» In both LP and MIP it is very important to distinguish between

- » known values
 - » data, parameters, etc.
- » and unknown values
 - » decision variables

» All constraints must be linear expressions of the variables



» Suggestion: name objects as follows

- » known values (data) using upper case
- » unknown values (variables) using lower case
- » constraints using mixed case

so that it is easy to distinguish between them, and see that constraints are indeed linear



- » Variables are actions that your model will prescribe
- » Use verbs for the names of variables
 - » this emphasizes that variables represent 'what to do' decisions
- » Indices are the objects that the actions are performed on
- » Use nouns for the names of indices



» Using named index sets/ranges

- » improves the readability of a model
- » makes it easier to apply the model to different sized data sets
- » makes the model easier to maintain
- » may speed up your model



» Try to include 'Min' or 'Max' in the name of your objective function
» An objective function called 'Obj' is not very helpful when taken out of context!



- » Comments are essential for a well written model
- » Always use a comment to explain what each parameter, data table, variable, and constraint is for when you declare it
- » Add extra comments to explain any complex calculation etc.

Model building style



» Comments in Mosel:

```
declarations
PRODUCTS = 1..NP ! Set of products
TIMES = 1..NT ! Set of time periods
make: array(PRODUCTS, TIMES) of mpvar
! Amount of p produced in time t
sell: array(PRODUCTS, TIMES) of mpvar
! Amount of p sold in time t
end-declarations
```

(! And here is a multi-line comment !) forall(t in TIMES)



Accessing data sources

» The initializations block
» Dynamic arrays
» Run-time parameters
» Using other data sources





- » Text files
- » ODBC
- » Sparse data

Separation of problem logic and data

- » Typically, the model logic stays constant once developed, with the data changing each run
 » Editing the model can create errors, expose intellectual property, and is impractical for industrial size data
- » It makes good sense to fix the model and obtain data from their source



uses "mmxprs"

declarations
PRODS = 1..2
DUR, WOOD, PROFIT: array(PRODS) of real ! Coefficients
x: array(PRODS) of mpvar
! Array of variables
end-declarations

initializations from "chess.dat" ! Read data from file
DUR WOOD PROFIT ! chess.dat: PROFIT: [5 20]
end-initializations ! DUR: [3 2]
! WOOD: [1 3]

sum(i in PRODS) DUR(i)*x(i) <= 160 ! Constraint definition
sum(i in PRODS) WOOD(i)*x(i) <= 200
forall(i in PRODS) x(i) is_integer
maximize(sum(i in PRODS) PROFIT(i)*x(i))
writeln("Solution: ", getobjval)</pre>

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» Every data item/table has a label, its identifier
 » Single line comments (marked with '!')

! Data file for 'chess4.mos'

DUR: [3 2] WOOD: [1 3] PROFIT: [5 20]



 » Every data entry specified with its index tuple
 » Can read data from one labeled data source into several Mosel data tables at once
 » data tables must have identical indices

initializations from 'chess.dat'
[DUR, WOOD, PROFIT] as 'ChessData'
end-initializations



» Format of data file with several data values in one labeled data range (use a * for a missing data value)

! chess.dat

ChessData: [(1) [3 1 5](2) [2 3 20]

» You can write out values in an analogous way to reading them in using initializations to

» To write out the solution values of variables, or other solution values (slack, activity, dual, reduced cost) you must first put the values into a data table

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```
declarations
  x_sol: array(PRODS) of real
end-declarations
```

```
forall(i in PRODS)
  x_sol(i) := getsol(x(i))
```

```
initializations to 'result.dat'
  x_sol
end-initializations
```



fopen("result.dat", F_OUTPUT+F_APPEND)

forall(i in PRODS)
writeln(i, ": ", getsol(x(i))

fclose(F_OUTPUT)



- » Modify the model chess4.mos to use indices of type string.
- » Execute this new model chess4s.mos with data set chess2.dat.
- » Output the solution values to file sol.dat using initializations to.
- » Modify the models further to read the contents of the index set from file (chess5.mos, chess5s.mos).



- Mosel provides a user friendly and efficient means of modeling mathematical programming problems
- » Objects such as dynamic arrays and variable index sets, together with efficient loops and sums, allow large scale models to be written easily, and execute quickly



- » Dynamic array: indexing sets not known at declaration, or array explicitly marked dynamic
- » Initialize dynamic data arrays from text files or using ODBC
 - » data must use sparse format
 - » this is so Mosel can work out the values of the indices
 - » reading in the data array initializes both the index values and the data values at the same time



 An entry of a dynamic array is only created when a value is assigned to it
 Decision variables don't get created, because you don't assign values to them
 To create decision variables in a dynamic array, use the create procedure

Dynamic arrays of **decision** variables



```
declarations
TIME: range ! = set of contiguous integers
COST: array(TIME) of real
use: array(TIME) of mpvar
end-declarations
(...) ! Read in COST data etc
forall(t in TIME | exists(COST(t)))
create(use(t))
```



 Note: if you declare decision variables after reading in the data, then decision variables will be created for all combinations of the index set elements that exist at that time
 Do not use create in this case
 Define decision variables before reading in data if you want to use create to control exactly which elements get created



» Use dynamic arrays

- » to size data tables automatically when the data is read in
- » to initialize the index values automatically when the data is read in
- » to conserve memory when storing sparse data
- » to eliminate index combinations without using conditions each time



» Don't use dynamic arrays

» when you can use an ordinary (static) array instead

» when storing dense data, and you can size the data table and initialize the indices in some other way (dynamic arrays are slower and use more memory than a static array when storing dense data)

Data input from file: Chess 4 completed

uses "mmxprs" parameters FILENAME="chess.dat" end-parameters

declarations
PRODS = 1..2
DUR, WOOD, PROFIT: array(PRODS) of real ! Coefficients
x: array(PRODS) of mpvar
! Array of variables
end-declarations

initializations from FILENAME DUR WOOD PROFIT end-initializations

sum(i in PRODS) DUR(i)*x(i) <= 160 ! Constraint definition
sum(i in PRODS) WOOD(i)*x(i) <= 200
forall(i in PRODS) x(i) is_integer
maximize(sum(i in PRODS) PROFIT(i)*x(i))</pre>

! Name of the data file

! Read data from file



Run-time parameters



» Parameters

- » a special type of constant
- » default value may be overriden at run-time

parameters
DATA_DIR = 'c:/data'
DEBUG = true
NUM_RECORDS = 1000
end-parameters



- » The value in the model is used by default
- » A different value may be given at run-time
 - » In IVE, an alternative value may be set in the Build \gg Options dialogue
 - » When running a Mosel model from an application, an alternative value can be set in the parameters string



» A parameters section must come at the top of the model

- » after any uses or options statements
- » before any other statements



- » Parameters are especially useful for passing directories/paths into the model
 - » all files referenced in the model should use a directory parameter
 - » otherwise, Mosel may not be able to find the file when the model is deployed (the default path differs when run from an application)
 - » use '+' to join strings



» Specifying directory paths » preferably use '/' as directory separator

parameters
DIR = '.'
end-parameters

fopen(DIR+'/cap.dat', F_INPUT)

```
fclose(F_INPUT)
```

. . .

initializations from DIR+'/cost.dat'



- » In models chess5.mos and chess5s.mos turn the data file name into a run-time parameter.
- » Re-run your model chess5s.mos with the larger data set chess3.dat without changing the filename in the model.



» Setting runtime parameters within IVE:

- » select menu Build \gg Options or click on the button \square
- » check Use model parameters to activate the parameter input field and enter the new value(s)

» Runtime parameters from the command line:

mosel -c "exe chess5s.mos DATAFILE='chess3.dat'"

» or:

```
mosel
exe chess5s.mos DATAFILE='chess3.dat'
quit
```



- The initializations block can work with many different data sources and formats thanks to the notion of I/O drivers
 I/O drivers for physical data files: mmodbc.excel, mmoci.oci, mmetc.diskdata
 Other drivers available, e.g. for data exchange in memory
- » Change of the data source = change of the I/O driver, no other modifications to your model



- » First, must check ODBC driver for your chosen data source (external to Xpress)
 - » Start ≫ Settings ≫ Control Panel ≫ Administrative
 Tools ≫ Data Sources (ODBC)
 - » Check that data source is defined, and note its name (the data source name, DSN)



» Next, identify specific data source – a database or spreadsheet

» note its location (path)

» the data must be in a table in a database, or a named range in a spreadsheet



» Now, in your model

- » use the *mmodbc* module (requires licence)
- » use the odbc driver in initializations blocks, or
- » write out the corresponding SQL commands:
 - » set up an ODBC data connection to the specific data source
 - » input data using SQL statements
 - » disconnect



» Excel spreadsheet ('ChessData' = range in the spreadsheet):

initializations from 'mmodbc.odbc:chess.xls'
[DUR, WOOD, PROFIT] as 'ChessData'
end-initializations

» Access database ('ChessData' = data table):

initializations from 'mmodbc.odbc:debug;chess.mdb'
[DUR, WOOD, PROFIT] as 'ChessData'
end-initializations



initializations to 'mmodbc.odbc:debug;chess.mdb'
 x_sol as 'ChessSol'
end-initializations

 » Before every new run, delete the data from the previous run in the destination range/table
 » Otherwise the new results will either be appended to the existing ones or, if 'PRODS' has been defined as key field in a database, the insertion will fail

Special notes for data export to Excel

- Make sure the 'Read Only' option is disabled in the ODBC data source set-up options
 Define the destination range in the
- spreadsheet, with one line of column headings, one line of dummy data, and no other data
- » Excel does not support the full range of ODBC functionality (commands like 'update' or 'delete' will fail)
 ⇒ preferably use direct connection (excel driver)



» Software-specific driver excel for MS Excel

- » use mmodbc module (requires licence)
- » use the excel driver (instead of odbc) in initializations blocks
- » no driver setup required (works with standard Excel installation)
- » simply replace "mmodbc.odbc:" by "mmodbc.excel:skiph;" in the preceding examples



» Software-specific driver oci for Oracle databases

- » use *mmoci* module (requires licence)
- » setup: Oracle's Instant Client package must be installed on the machine running the Mosel model
- » in initializations blocks replace
 "mmodbc.odbc:" by "mmoci.oci:" in the

preceding examples

» supports SQL statements (replace the prefix SQL by OCI)





- » The I/O driver odbc generates automatically the SQL commands required to connect to the database/spreadsheet
- » For advanced uses module *mmodbc* also defines most standard SQL commands directly for the Mosel language



- » Check that the ODBC DSN for Excel is set up on your computer
 » Re-run your model chess5.mos with the Excel
 - file chess.xls



» We have seen that it is possible to completely separate the data and the model
» The model specifies the logic of the problem, without any reference to its size
» The model can be applied to any data instance, simply by providing data files



» Refer to the Mosel User Guide for a detailed introduction to working with Mosel.

» The book Applications of optimization with Xpress-MP provides a large collection of examples models from different application areas.

» See the whitepaper Using ODBC and other database interfaces with Mosel for further detail on data handling.



Advanced modeling topics

» MIP variable types» Modeling with binary variables





» MIP variable types» Modeling with binary variables

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» Binary variables

- » can take either the value 0 or the value 1 (do/ don't do variables)
- » model logical conditions

x(4) is_binary

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» Integer variables

- » can take only integer values
- used where the underlying decision variable really has to take on a whole number value for the optimal solution to make sense

x(7) is_integer



» Partial integer variables

- » can take integer values up to a specified limit and any value above that limit
- » computational advantages in problems where it is acceptable to round the LP solution to an integer if the optimal value of a decision variable is quite large, but unacceptable if it is small

x(1) is_partint 5 ! Integer up to 5, then continuous



» Semi-continuous variables

- » can take either the value 0, or a value between some lower limit and upper limit
- » help model situations where if a variable is to be used at all, it has to be used at some minimum level

x(2) is_semcont 6 ! A 'hole' between 0 and 6, then continuous



» Semi-continuous integer variables

- » can take either the value 0, or an integer value
 between some lower limit and upper limit
- » help model situations where if a variable is to be used at all, it has to be used at some minimum level, and has to be integer

x(3) is_semint 7 ! A 'hole' between 0 and 7, then integer



» Special Ordered Sets of type one (SOS1)

- » an ordered set of variables at most one of which can take a non-zero value
- » single choice among several possibilities

» Special Ordered Sets of type two (SOS2)

- » an ordered set of variables, of which at most two can be non-zero, and if two are non-zero these must be consecutive in their ordering
- » e.g. approximation of non-linear functions with a piecewise linear function



» WEIGHT array determines the ordering of the variables:

MYSOS:= sum(i in IRng) WEIGHT(i) *x(i) is_sosX

where is_sosX is either is_sos1 or is_sos2



» Alternative: set S of set members, linear constraint L with ordering coefficients (= reference row entries):

makesos1(S,L); makesos2(S,L)

» must be used if the coefficient WEIGHT(i) of an intended set member is zero

» Note: the ordering coefficients must all be distinct (or else they are not doing their job of supplying an order!)



- » Projects A, B, C, D
- » Binary variables a, b, c, d
 - » do at most 3 projects: $a + b + c + d \le 3$
 - » must do D if A done: $d \ge a$
 - » can only do C if both A and B done:
 - $c \le (a + b) / 2$
 - $c \leq a, c \leq b$





» Either

 $5 \le x \le 10$

or

 $80 \le x \le 100$

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» Introduce a new variable: ifupper: 0 if $5 \le x \le 10$; 1 if $80 \le x \le 100$ $x \le 10 + (100 - 10) \cdot ifupper$ [1] $x \ge 5 + (80 - 5) \cdot ifupper$ [2]





» Either $5 \le \sum_i A_i x_i \le 10$ or $80 \le \sum_i A_i x_i \le 100$

 $\sum_{i} A_{i} x_{i} \leq 10 + 90 \cdot ifupper$ $\sum_{i} A_{i} x_{i} \geq 5 + 75 \cdot ifupper$



» Two variables

*x*₁, *x*₂

with

 $0 \leq x_i \leq U$ [1. *i*]

want

 $y = |x_1 - x_2|$

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» Introduce binary variables

 d_1, d_2

to mean d_1 : 1 if $x_1 - x_2$ is the positive value d_2 : 1 if $x_2 - x_1$ is the positive value



» MIP formulation of $y = |x_1 - x_2|$ $0 \le x_i \le U$ [1.i] $0 \le y - (x_1 - x_2) \le 2 \cdot U \cdot d_2$ [2] $0 \le y - (x_2 - x_1) \le 2 \cdot U \cdot d_1$ [3] $d_1 + d_2 = 1$ [4]



» Take a look at the capital budgeting model in capbgt.mos: the objective is to determine the most profitable choice among 8 possible projects, subject to limited resources (personnel and capital)

- » Formulate the following additional constraints:
 - » P1 can only be done if P2 is done
 - » P1 can only be done if P3 and P6 are done
 - » It is not possible to do both P5 and P6
 - » Either P1 and P2 must be done or P3 and P4 (but not both pairs).

Solution



! p1 can only be done if p2 is done
x(2) >= x(1)

! p1 can only be done if p3 and p6 are done $(x(3) + x(6))/2 \ge x(1)$

! It is not possible to do both p5 and p6 $x(5) + x(6) \le 1$

! Either p1 and p2 must be done or p3 and p4 (but not both pairs). x(1) = x(2); x(3) = x(4)x(1) + x(2) = 2 - (x(3) + x(4))



Programming language features

- » Selections
- » Loops
- » Functions and procedures
- » Data structures
- » Programming solution algorithms

Mosel: A programming environment

FICO

- » Selections
- » Loops
- » Set operations
- » Subroutines
- » Data structures

Selections



» if

if A >= 20 then
 x <= 7
elif A <= 10 then
 x >= 35
else
 x = 0
end-if

» case

```
case A of
  -1000..10 : x >= 35
  20..1000 : x <= 7
  12, 15 : x = 1
  else x = 0
end-case
```





- » forall [do]
- » while [do]
- » repeat until



» Implements the 'Sieve of Eratosthenes'.

```
SNumbers = \{2, ..., L\}

n := 2

repeat

while (n \not\in SNumbers) n := n + 1

SPrime := SPrime \cup \{n\}

i := n

while (i \le L)

SNumbers := SNumbers \setminus \{i\}

i := i + n

until SNumbers = \{\}
```



model Prime
parameters
LIMIT=100 ! Search for prime numbers in 2..LIMIT
end-parameters

declarations
SNumbers: set of integer ! Set of numbers to be checked
SPrime: set of integer ! Set of prime numbers
end-declarations

SNumbers:={2..LIMIT}
writeln("Prime numbers between 2 and ", LIMIT, ":")

Example: Prime numbers



```
n:=2
repeat
while (not(n in SNumbers)) n+=1
SPrime += {n} ! n is a prime number
i:=n
while (i<=LIMIT) do ! Remove n and all its multiples
SNumbers-= {i}
i+=n
end-do
until SNumbers={}</pre>
```

```
writeln(SPrime)
writeln(" (", getsize(SPrime), " prime numbers.)")
end-model
```



» Set operators include

- » union: +
- » intersection: *
- » difference: -

» Logical expressions using sets include

- » subset: Set1 <= Set2</pre>
- » superset: Set1 >= Set2
- » equals: Set1 = Set2
- » not equals: Set1 <>Set2
- » element of: 'Oil5' in Set1
- » not element of: 'Oil5' not in Set1



» Similar structure as model, including the declarations blocks

» Terminated by end-function or end-procedure

- » Function defines returned with its return value
- » forward declaration
- » Overloading possible (each version with a different number or types of arguments)



- Choose a middle value v for partitioning (here: v = (min + max) / 2)
- Divide the list into two parts 'left' (all elements x < v) and 'right' (all elements x > v)
- 3. Repeat from 1. for lists 'left' and 'right'



```
model "Quick Sort"
parameters
LIM=50
end-parameters
```

! Declare procedures that are defined later forward procedure qsort(L:array(range) of integer) forward procedure qsort(L:array(range) of integer, s,e:integer)

```
declarations
T:array(1..LIM) of integer
end-declarations
```

```
! Generate randomly an array of numbers
forall(i in 1..LIM) T(i):=round(.5+random*LIM)
writeln(T)
time:=gettime
```

qsort(T)	! Sort the array
writeln(T)	! Print the sorted array



! Swap the positions of two numbers in an array
procedure swap(L:array(range) of integer, i,j:integer)
k:=L(i)
L(i):=L(j)
L(j):=k
end-procedure

! Start of the sorting process
procedure qsort(L:array(r:range) of integer)
qsort(L,getfirst(r),getlast(r))
end-procedure

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```
! Sorting routine
procedure qsort(L:array(range) of integer, s,e:integer)
 v := L((s+e) \operatorname{div} 2)
 i:=s; j:=e
 repeat
  while (L(i) < v) i+=1
  while (L(j) > v) j = 1
   if i<j then
   swap(L,i,j)
   i+=1; j-=1
   end-if
 until i>=j
 if j<e and s<j then qsort(L,s,j); end-if
 if i>s and i<e then qsort(L,i,e); end-if
end-procedure
```

end-model



» array

» set

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- » array
- » set
- » list
- » record



- » array
- » set
- » list
- » record

» ... and any combinations thereof, e.g.,

S: set of list of integer
A: array(range) of set of real





- » Collection of objects of the same type
- » May contain the same element several times
- » Order of list elements is specified by construction
- » Handling: cuthead, splittail, reverse...

```
declarations
  L: list of integer
  M: array(range) of list of string
  end-declarations
```

```
L:= [1,2,3,4,5]
M:: (2..4)[['A','B','C'], ['D','E'], ['F','G','H','I']]
```



- » Finite collection of objects of any type
- » Each component of a record is called a 'field' and is characterized by its name and its type

```
declarations
```

```
ARC: array (ARCSET: range) of record
```

Source, Sink: string

Cost: real

end-record end-declarations ! Source and sink of arc
! Cost coefficient

ARC(1).Source:= "B" ARC(3).Cost:= 1.5



 » Treated in the same way as the predefined types of the Mosel language
 » New types are defined in declarations blocks by specifying a type name, followed by =, and the definition of the type

```
declarations
  myreal = real
  myarray = array(1..10) of myreal
  COST: myarray
end-declarations
```

User types



» Typical uses

» shorthand for repetitions in declarations

! Source and sink of arc

! Cost coefficient

» naming records

declarations
arc = record
Source,Sink: string
Cost: real
end-record
A: arc

ARC: array(ARCSET:range) of arc

end-declarations



- » Data structures: array, set, list, record
- » Selections: if-then-[elif-then]-[else], case
- » Loops: forall-[do], while-[do], repeat-until
- » Operators:
 - » standard arithmetic operators
 - » aggregate operators (sum, prod, and, or, min, max, union, intersection)
 - » set operators
- » Subroutines: functions, procedures (forward declaration, overloading)



» No separation between 'modeling statements' and 'solving statements'

- » Programming facilities for pre/postprocessing, algorithms
- » Principle of incrementality
- » Not solver-specific
- » Possibility of interaction with solver(s)



» Solution heuristic written with Mosel» Program split into several source files

Solving: Variable fixing heuristic (main file)

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model Coco
uses "mmxprs"

include "fixbv_pb.mos"
include "fixbv_solve.mos"

solution:=solve
writeln("The objective value is: ", solution)

end-model

Solving: Variable fixing heuristic (model)

```
declarations
RF=1..2 ! Range of factories (f)
RT=1..4 ! Range of time periods (t)
(...)
openm: array(RF,RT) of mpvar
end-declarations
(...)
forall(f in RF,t in 1..NT-1) Closed(f,t):= openm(f,t+1) <= openm(f,t)</pre>
```

FICO

```
forall(f in RF,t in RT) openm(f,t) is_binary
```

Solving: Variable fixing heuristic (algorithm)

function solve:real declarations osol: array(RF,1..2) of real bas: basis end-declarations

setparam("XPRS_PRESOLVE", 0) setparam("zerotol", 5.0E-4) ! Set Mosel comparison tolerance maximize (XPRS LPSTOP, MaxProfit) ! Solve the root LP savebasis(bas)

! Save the basis

FICO

```
forall(f in RF, t in 1..2) do ! Fix some binary variables
osol(f,t) := getsol(openm(f,t))
 if osol(f,t) = 0 then
  setub(openm(f,t), 0.0)
elif osol(f,t) = 1 then
  setlb(openm(f,t), 1.0)
end-if
end-do
```

Solving: Variable fixing heuristic (algorithm)

```
maximize(XPRS CONT, MaxProfit) ! Solve modified problem
solval:=getobjval
```

! Save solution value

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```
forall(f in RF, t in 1..2) ! Reset variable bounds
 if((osol(f,t) = 0) \text{ or } (osol(f,t) = 1)) \text{ then}
 set1b(openm(f,t), 0.0)
  setub(openm(f,t), 1.0)
 end-if
```

loadbasis(bas) ! Load previously saved basis setparam("XPRS_MIPABSCUTOFF", solval) ! Set cutoff value maximize(MaxProfit) ! Solve original problem returned:= if (getprobstat=XPRS_OPT, getobjval, solval) end-function



Mosel modules and packages



Mosel: A modular environment



» Open architecture:

 » possibility to define language extensions via packages or modules without any need to modify the core of the Mosel language



» Package = library written in the Mosel language

- » making parts of Mosel models re-usable
- » deployment of Mosel code whilst protecting your intellectual property
- » similar structure as models (keyword model is replaced by package), compiled in the same way
- » included with the uses statement
- » definition of new types, subroutines, symbols
- » see examples in the Mosel User Guide



» Module = dynamic library written in C

- » modules of the Mosel distribution:
 - » solver interfaces:
 - Xpress-Optimizer (LP, MIP, QP), SLP, SP, CP
 - » database access: ODBC, OCI
 - » system commands; model handling; graphics
- » write your own modules for
 - » connecting to external software
 - » time-critical tasks
 - » defining new types, subroutines, operators, I/O drivers, control parameters, symbols



 Interaction with external programs during their execution (callback functions)

- Access to other solvers and solving paradigms (NLP, CP)
- » Implementation of graphical applications (*mmive*, XAD)



```
uses "mmxprs"
```

```
declarations
  x: array(1..10) of mpvar
end-declarations
```

```
public procedure printsol
writeln("Solution: ", getsol(Objective))
forall(i in 1..10) write("x(", i, ")=", getsol(x(i)), "")
writeln
end-procedure
```

setcallback(XPRS_CB_INTSOL, "printsol")



» What is the greatest area of a polygon of N sides and a diameter of 1?



model "Polygon"
uses "mmxslp"

declarations
N=5
area: gexp
rho, theta: array(1..N) of mpvar
objdef: mpvar
D: array(1..N,1..N) of genctr
end-declarations

```
forall(i in 1..N-1) do ! Initialization of SLP variables
rho(i) >= 0.1; rho(i) <= 1
SLPDATA("IV", rho(i), 4*i*(N + 1 - i)/((N+1)^2))
SLPDATA("IV", theta(i), M_PI*i/N)
end-do</pre>
```

Module mmxslp: Solving an NLP by SLP



```
! Vertices in increasing order
forall(i in 2...N-1) theta(i) >= theta(i-1) +.01
```

```
theta(N-1) <= M_PI
```

! Boundary conditions

area:=

! Objective: sum of areas

```
(sum(i in 2..N-1) (rho(i)*rho(i-1)*sin(theta(i)-theta(i-1))))*0.5
objdef = area; objdef is_free
SLPloadprob(objdef)
SLPmaximize
```

```
writeln("Area = ", getobjval)
end-model
```



» Example: jobshop scheduling

- » schedule the production of a set of jobs on a set of machines. Every job is produced by a sequence of tasks, each of these tasks is processed on a different machine. A machine processes at most one job at a time.
- » Implementation with high-level modeling objects (tasks and resources)

Module kalis: **Constraint Programming**



model "Job Shop" uses "kalis"

declarations

JOBS = 1..NJMACH = 1..NMRES: array (JOBS, MACH) of integer DUR: array (JOBS, MACH) of integer

res: array (MACH) of cpresource task: array(JOBS, MACH) of cptask ! Set of jobs ! Set of resources ! Resource use of tasks ! Durations of tasks

end-declarations

! Resources ! Tasks

! Initialize the data . . .

HORIZON:= sum(j in JOBS, m in MACH) DUR(j,m)

Module kalis: Constraint Programming



forall(j in JOBS) getend(task(j,NM)) <= HORIZON</pre>

```
! Setting up the resources (capacity 1)
forall(m in MACH)
set resource attributes(res(m), KALIS UNARY RESOURCE, 1)
```

! Setting up the tasks (durations, resource used)
forall(j in JOBS, m in MACH)
set_task_attributes(task(j,m), DUR(j,m), res(RES(j,m)))

! Precedence constraints between the tasks of every job forall (j in JOBS, m in 1..NM-1) setsuccessors(task(j,m), {task(j,m+1)})

! Solve the problem & print solution if cp_schedule(getmakespan) <>0 then writeln("Total completion time: ", getsol(getmakespan)) end-if end-model

Module mmive: Drawing user graphs



```
model "Schedule"
uses "mmive", "mmsystem"
```

```
declarations
MACHINES=6; JOBS=6
graphs, colors: array(1..MACHINES) of integer
labels: array(1..JOBS) of integer
curmachine, curjobs, n1, n2, n3: integer
end-declarations
```

```
forall (i in 1..MACHINES) do
graphs(i) := IVEaddplot("Machine "+i, IVE_BLUE)
labels(i) := IVEaddplot("Jobs for machine "+i, Color(i))
end-do
```

```
FICO
```

end-do end-do

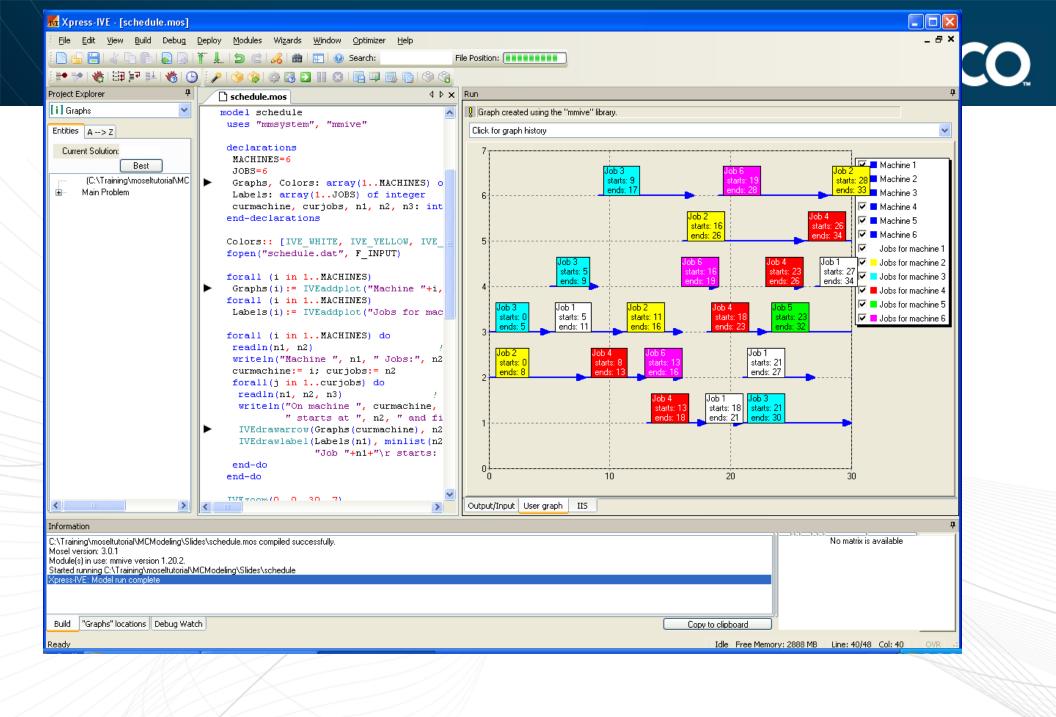
IVEzoom(0, 0, 30, 7 fclose(F INPUT)

end-model

Module mmive: Drawing user graphs



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- » Working with several models in parallel, possibly in a heterogeneous distributed architecture (module *mmjobs*)
 - » see whitepaper Multiple models and parallel solving with Mosel
- » Combining different solvers
 - » see whitepaper Hybrid MIP/CP solving with Xpress-Optimizer and Xpress-Kalis



 The modules of the Mosel distribution are documented in the Mosel Language Reference Manual (with separate manuals for solver modules *mmxslp* and *kalis*)
 The Mosel Native Interface User Guide explains how to write your own modules.

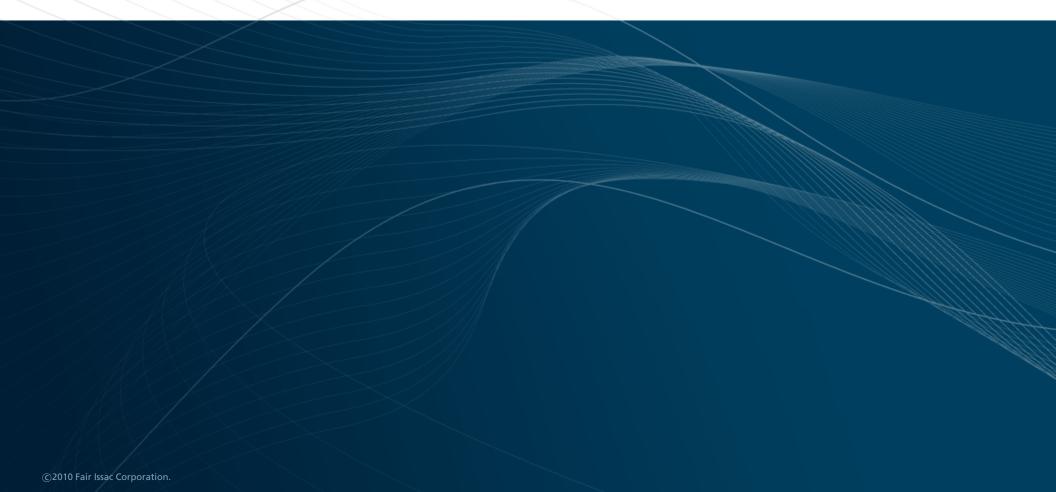


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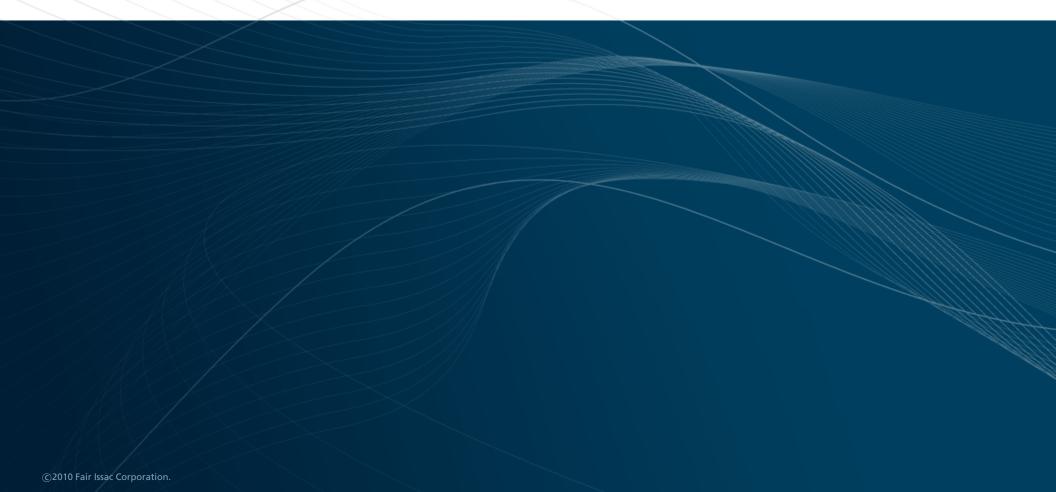


Embedding Mosel models





Embedding models in applications





The Mosel language allows you to formulate optimization problems, and develop optimization methods (*i.e.*, use the Optimizer to solve them), as a Mosel model
 The Mosel API (also Mosel libraries) allows you to embed Mosel models in an application



- » The Mosel API is available for C/C++, Java, .NET and VB
- » We use Java in the slides, but the functionality applies to all languages, and similar applications can be developed in other languages



» Model Compiler Library

 » compiles to a virtual machine
 » binary format architecture independent

 » Runtime Library

 » load and run binary (models)
 » access to Mosel internal database (data, solution values, ...)



» With Xpress-IVE: select Deploy >> Deploy or click the deploy button <>>>

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» Choose the application language:

Deployment		×
The candidate file for deplo	oyment is:	
C:\Examples\chess5.mos		
How would you like to use	this Mosel model in your app	lication?
Save .BIM file	Run Mosel model from	Optimize matrix file from
O With debug info	O C	O C
C All names stripped	🖸 Java	C Java
Save .BIM file	C Visual Basic	C Visual Basic
	C VB.NET	C VB.NET
To directly create a Windows executable that runs a .BIM file:	○ C#	O C#
1. Copy C:WpressMP\bin	Nmrun.exe to the same folde	r as the .BIM file;
2. Rename mrun.exe to m	atch the name of the .BIM fil	e, but with .EXE instead.
1	lext >C	Cancel



» Clicking on the Next button will open a new window with the resulting code
» Use the Save as button to set the name and location of the new file.

Mosel library functions



» General:

XPRM(), XPRM.getVersion, XPRM.license, ...

» Model handling:

XPRM.compile, XPRM.loadModel, XPRMModel.run, XPRMmodel.getResult, XPRMModel.getExecStatus, XPRMModel.reset, ...

» Solution information:

XPRMModel.getObjectiveValue, XPRMModel.getProblemStatus, XPRMMPVar.getSolution, XPRMLinCtr.getActivity, ...

Mosel library functions



» Accessing model objects:

XPRMModel.findIdentifier

» Arrays:

XPRMArray.getDimension, XPRMArray.getIndexSets, XPRMArray.getFirstIndex, XPRMArray.nextIndex, XPRMArray.get, ...

» Sets:

XPRMSet.getSize, XPRMSet.getFirstIndex, XPRMSet.isFixed, ...

» Handling of modules:

XPRM.findModule, XPRM.setModulesPath, XPRMModule.parameters, ...



 » Use IVE to generate a Java program that compiles and runs model chess5.mos
 » Modify the program so that the model execution uses the data file chess4.dat.
 » Check the problem status and output the objective value.

Solution



```
import com.dashoptimization.*;
```

```
public class chessc
```

public static void main(String[] args) throws Exception

```
int result;
XPRMModel model;
XPRM xprm;
```

```
// Initialize Mosel
xprm = new XPRM();
```

// Load compiled model (.BIM file)
model = xprm.loadModel("chess5.bim");

Solution



// Check problem status and retrieve the optimal solution value
if (model.getProblemStatus() == XPRMModel.PB_OPTIMAL)
System.out.println("Objective value: " +

model.getObjectiveValue());

```
model.reset();
```



» Retrieving detailed solution information and model data

XPRMModel model; XPRMSet prods; XPRMArray profit, ax; XPRMMPVar x; int[] idx = new int[1]; double val;

// Retrieve solution values and problem data
prods = (XPRMSet)model.findIdentifier("PRODS");
profit = (XPRMArray)model.findIdentifier("PROFIT");
ax = (XPRMArray)model.findIdentifier("x");





» Data exchange in memory with host application

```
public class chessio
  static int NP = 4;
                                        // Input data
  static final double[] dur = \{3, 2, 2, 3\};
  static final double[] wood = \{1, 2, 3, 6\};
  static final double [] profit = \{5, 12, 20, 40\};
                                        // Array for solution values
  static double[] solution = new double[NP];
 public static void main(String[] args) throws Exception
    int result;
    XPRMModel model;
    XPRM xprm;
```

```
// Initialize Mosel
xprm = new XPRM();
xprm.compile("chess5ioj.mos");
                                   // Compile + load model
model = xprm.loadModel("chess5ioj.bim");
xprm.bind("DUR", dur);
                                   // Associate Java objects with
xprm.bind("WOOD", wood);
                                    // names in Mosel
xprm.bind("PROFIT", profit);
xprm.bind("xsol", solution);
model.execParams = "NP="+NP;
                                    // Set runtime parameters
model.run();
                                    // Run the model
if (model.getProblemStatus() == model.PB OPTIMAL)
                // Check problem status and display the solution
  System.out.println("Objective: " + model.getObjectiveValue());
  for(int i=0;i<NP;i++)</pre>
    System.out.println("x(" + (i+1) + "): " + solution[i] +
                        "\t (profit: " + profit[i] + ")");
model.reset();
```

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 Mosel libraries allow you to embed model programs directly in your application
 Access the solution directly in your application, as alternative to using ODBC
 Enjoy benefits of structured modeling language and rapid deployment when building applications



- » May choose to work with compiled models rather than model source files – provides protection against the user viewing / changing the model
- » Compiled models are platform independent



You will find it helpful to refer to the Mosel Libraries Reference Manual

» The part 'Working with the Mosel libraries' of the Mosel User Guide documents examples for different programming language interfaces

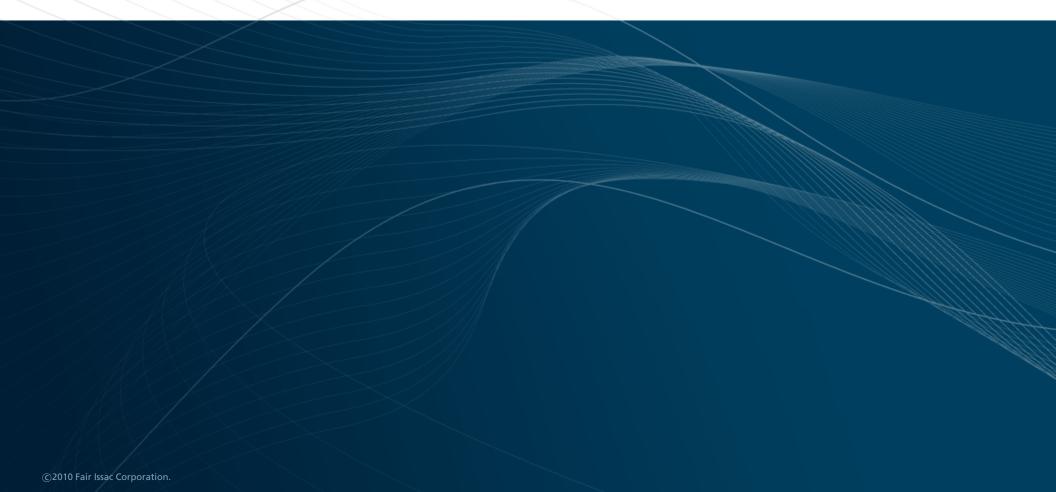


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Summary and further information





Summary

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» FICO Xpress product suite

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- » FICO Xpress product suite
 - » solvers





- » FICO Xpress product suite
 - » solvers
 - » modeling interfaces

Summary



- » FICO Xpress product suite
 - » solvers
 - » modeling interfaces
 - » development environment





» Modeling with Mosel

Formulating Linear and Mixed Integer Programming (LP and MIP) problems





- » Modeling with Mosel
 - » formulating Linear and Mixed Integer Programming (LP and MIP) problems
 - » accessing data sources





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 - » programming language features





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 - » language extensions (modules and packages)





- » Modeling with Mosel
 - » formulating Linear and Mixed Integer Programming (LP and MIP) problems
 - » accessing data sources
 - » programming language features
 - » language extensions (modules and packages)
- » Embedding models in applications for deployment



» Xpress website: http://www.fico.com/xpress » Examples database: http://examples.xpress.fico.com » Whitepapers, documentation: http://optimization.fico.com



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